OPERATEURS DIFFERENTIELS - FORMULAIRE

Cartésiennes (x,y,z)

$$\overrightarrow{\text{grad}}\, f = \vec{\nabla} f = \begin{vmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{vmatrix}$$

$$\text{div}\, \vec{F} = \vec{\nabla}.\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\overrightarrow{\text{rot}}\, \vec{F} = \nabla \wedge \vec{F} = \begin{vmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{vmatrix}$$

$$\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\overrightarrow{\Delta \vec{F}} = \Delta F_x \vec{e}_x + \Delta F_y \vec{e}_y + \Delta F_z \vec{e}_z$$

Cylindriques $(,\varphi\rho,z)$

$$\overrightarrow{\text{grad}}\, f = \begin{vmatrix} \frac{\partial f}{\partial \rho} \\ \frac{1}{\rho}\frac{\partial f}{\partial \varphi} \\ \frac{\partial f}{\partial z} \end{vmatrix}$$

$$\text{div}\, \vec{F} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z}$$

$$\overrightarrow{\text{rot}}\, \vec{F} = \begin{vmatrix} \frac{1}{\rho}\left[\frac{\partial F_z}{\partial \varphi} - \frac{\partial}{\partial z}(\rho F_\varphi)\right] \\ \frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \\ \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho F_\varphi) - \frac{\partial F_\rho}{\partial \varphi}\right] \end{vmatrix}$$

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Sphériques (r,θ,φ)

$$\overrightarrow{\text{grad}}\, f = \begin{vmatrix} \frac{\partial f}{\partial r} \\ \frac{1}{r}\frac{\partial f}{\partial \theta} \\ \frac{1}{r \sin\theta}\frac{\partial f}{\partial \varphi} \end{vmatrix}$$

$$\text{div}\, \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(F_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial F_\varphi}{\partial \varphi}$$

$$\overrightarrow{\text{rot}}\, \vec{F} = \begin{vmatrix} \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(F_\varphi \sin\theta) - \frac{\partial F_\theta}{\partial \varphi}\right] \\ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \varphi} - \frac{\partial}{\partial r}(r F_\varphi)\right] \\ \frac{1}{r}\left[\frac{\partial}{\partial r}(r F_\theta) - \frac{\partial F_r}{\partial \theta}\right] \end{vmatrix}$$

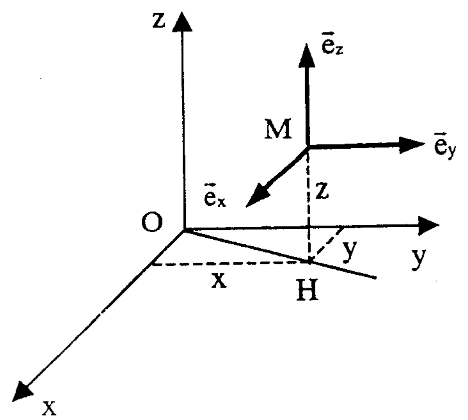
$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

$\overrightarrow{\text{grad}}\left(\text{div}\,\vec{F}\right) = \overrightarrow{\text{rot}}\left(\overrightarrow{\text{rot}}\,\vec{F}\right) + \overrightarrow{\Delta \vec{F}}$	$\text{div}\left(\overrightarrow{\text{grad}}\, f\right) = \Delta f$ $\text{div}\left(\overrightarrow{\text{rot}}\,\vec{F}\right) = 0$	$\overrightarrow{\text{rot}}\left(\overrightarrow{\text{grad}}\, f\right) = \vec{0}$ $\overrightarrow{\text{rot}}\left(\overrightarrow{\text{rot}}\,\vec{F}\right) = \overrightarrow{\text{grad}}\left(\text{div}\,\vec{F}\right) - \overrightarrow{\Delta \vec{F}}$
$\overrightarrow{\text{grad}}\,(f+g) = \overrightarrow{\text{grad}}\, f + \overrightarrow{\text{grad}}\, g$ $\overrightarrow{\text{grad}}\,(f\,g) = f\,\overrightarrow{\text{grad}}\, g + g\,\overrightarrow{\text{grad}}\, f$ $\overrightarrow{\text{grad}}\,(\vec{F}.\vec{G}) = \left(\vec{F} \wedge \overrightarrow{\text{rot}}\,\vec{G}\right) + \left(\vec{G} \wedge \overrightarrow{\text{rot}}\,\vec{F}\right) + \left(\vec{G}.\overrightarrow{\text{grad}}\right)\vec{F} + \left(\vec{F}.\overrightarrow{\text{grad}}\right)\vec{G}$	$\text{div}\left(\vec{F}+\vec{G}\right) = \text{div}\,\vec{F} + \text{div}\,\vec{G}$ $\text{div}\,(f\,\vec{F}) = f\,\text{div}\,(\vec{F}) + \vec{F}.\overrightarrow{\text{grad}}\, f$ $\text{div}\,(\vec{F} \wedge \vec{G}) = \vec{G}.\overrightarrow{\text{rot}}\,\vec{F} - \vec{F}.\overrightarrow{\text{rot}}\,\vec{G}$	$\overrightarrow{\text{rot}}\left(\vec{F}+\vec{G}\right) = \overrightarrow{\text{rot}}\,\vec{F} + \overrightarrow{\text{rot}}\,\vec{G}$ $\overrightarrow{\text{rot}}\left(f\,\vec{F}\right) = f\,\overrightarrow{\text{rot}}\,\vec{F} - \left(\vec{F} \wedge \overrightarrow{\text{grad}}\, f\right)$ $\overrightarrow{\text{rot}}\left(\vec{F} \wedge \vec{G}\right) = \vec{F}\,\text{div}\,\vec{G} - \vec{G}\,\text{div}\,\vec{F} + \left(\vec{G}.\overrightarrow{\text{grad}}\right)\vec{F} - \left(\vec{F}.\overrightarrow{\text{grad}}\right)\vec{G}$

SYSTEMES DE COORDONNEES

Cartésiennes (x,y,z)

Base locale : $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$



Position :

$$\overrightarrow{OM} = \vec{r} = x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z$$

Déplacement élémentaire :

$$\vec{dl} = dx\,\vec{e}_x + dy\,\vec{e}_y + dz\,\vec{e}_z$$

Surfaces élémentaires :

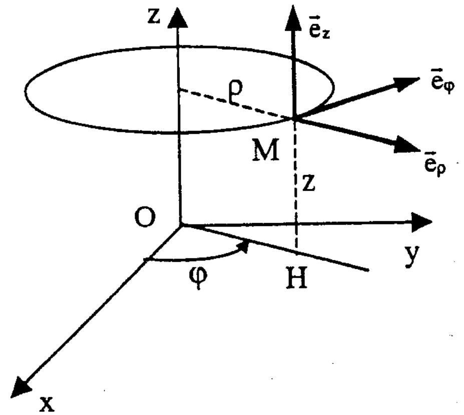
$dS_x = dy\,dz \quad dS_y = dx\,dz \quad dS_z = dx\,dy$

Volume élémentaire :

$$d\tau = dx\,dy\,dz$$

Cylindriques (ρ,φ,z)

Base locale $(\vec{e}_\rho, \vec{e}_\varphi, \vec{e}_z)$



Position :

$$\overrightarrow{OM} = \vec{r} = \rho\,\vec{e}_\rho + z\,\vec{e}_z$$

Déplacement élémentaire :

$$\vec{dl} = d\rho\,\vec{e}_\rho + \rho\,d\varphi\,\vec{e}_\varphi + dz\,\vec{e}_z$$

Surfaces élémentaires :

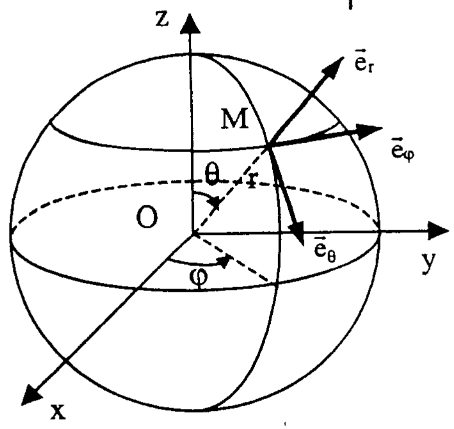
$dS_\rho = \rho\,d\varphi\,dz$; $dS_\varphi = d\rho\,dz$; $dS_z = \rho\,d\rho\,d\varphi$

Volume élémentaire :

$$d\tau = \rho\,d\rho\,d\varphi\,dz$$

Sphériques (r,θ,φ)

Base locale $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi)$



Position :

$$\overrightarrow{OM} = \vec{r} = r\,\vec{e}_r$$

Déplacement élémentaire :

$$\vec{dl} = dr\,\vec{e}_r + r\,d\theta\,\vec{e}_\theta + r\sin\theta\,d\varphi\,\vec{e}_\varphi$$

Surfaces élémentaires :

$dS_r = r^2\sin\theta\,d\theta\,d\varphi$; $dS_\theta = r\sin\theta\,dr\,d\varphi$;
$dS_\varphi = r\,dr\,d\theta$

Volume élémentaire :

$$d\tau = r^2\sin\theta\,dr\,d\theta\,d\varphi$$